it is shown in Ref. 5 that solutions are obtained when

$$\lambda = n \qquad \lambda = -n \qquad \lambda = 2 + n \qquad \lambda = 2 - n \tag{1}$$

Because of the double roots which occur when $n = 0$ or $n = 1$, the solutions for $F(r)$ are shown to be of the following three forms:

$$\left.\begin{aligned} n &= 0 & F(r) &= a_0 + b_0 \ln r + c_0 r^2 + d_0 r^2 \ln r \\ n &= 1 & F(r) &= a_1 r + b_1/r + c_1 r^3 + d_1 r \ln r \\ n &\geqslant 2 & F(r) &= a_n r^n + b_n r^{-n} + c_n r^{2+n} + d_n r^{2-n} \end{aligned}\right\} \tag{2}$$

The stress functions obtained from these expressions do not include the following terms which appear in Timoshenko's "general solution":

$$\Phi_1 = d_0 r^2 \theta \qquad \Phi_2 = a_0' \theta$$
$$\Phi_3 = (a_1/2) r\theta \sin\theta \qquad \Phi_4 = -(c_1/2) r\theta \cos\theta \tag{3}$$

In an unpublished M.S. thesis, Zuercher,[6] utilizing the same separation of variables approach as was later used in Ref. 5, showed that it is not sufficient to consider the multiple values of λ in Eq. (1) which occur when n takes on the values $n = 0$ and $n = 1$. To get a complete solution, it is also necessary to consider the multiple values of n which occur when λ takes on the values $\lambda = 0$ and $\lambda = 1$. This yields, in addition to all the terms in Eq. (2), the terms which lead to the stress function given in Eq. (3) plus the four "new" terms given by Sadeh.[1]

Zuercher obtained additional solutions to the biharmonic equation by assuming a separation of variables solution in the form

$$\Phi = F_1(r)G_1(\theta) + F_2(r)G_2(\theta)$$

where it was not necessary for each term itself to be biharmonic. In particular he considered

$$\Phi = r^n \ln r \begin{Bmatrix} \cos a\theta \\ \sin a\theta \end{Bmatrix} + r^n \theta \begin{Bmatrix} \sin a\theta \\ \cos a\theta \end{Bmatrix}$$

and this led to solutions of the form

$$\left.\begin{aligned} \Phi_1 &= f_n[r^n \ln r \cos n\theta - r^n\theta \sin n\theta] \\ \Phi_2 &= g_n[r^n \ln r \cos(n-2)\theta - r^n\theta \sin(n-2)\theta] \\ \Phi_3 &= h_n[r^n \ln r \sin n\theta + r^n\theta \cos n\theta] \\ \Phi_4 &= j_n[r^n \ln r \sin(n-2)\theta + r^n\theta \cos(n-2)\theta] \end{aligned}\right\} \tag{4}$$

where, except for the special cases of $n = 0$ and $n = 1$, each expression in brackets must be taken in its entirety. With regard to the physical significance of any of these "new" terms, Hyman[7] showed that the stress functions for three problems listed in Timoshenko (problems 17, 20, and 21 of Chap. 4) when written in polar form contain terms that are included in Eq. (4).

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Comment on "A Note on the General Solution of the Two-Dimensional Linear Elasticity Problem in Polar Coordinates"

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THE additional solutions to the two-dimensional biharmonic equation in polar coordinates found by W. Z. Sadeh in Ref. 1 have been known for several decades! Applications of these solutions to slit plates and ring plate sectors can be found in Refs. 2, 3, and elsewhere.

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Reply by Author to the Comments by C. W. Bert, B. I. Hyman, and F. Y. M. Wan

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THE author is indebted to C. W. Bert, B. I. Hyman, and Y. M. Wan for their useful and very pertinent comments. Unfortunately it was only after the publication of the Technical Note that the author became aware of an earlier similar solution by Filonenko-Borodich,[1] through a private communication from P. H. Francis (Senior Research Engineer, Department of Mechanical Sciences, Southwest Research Institute, San Antonio, Texas). In addition to this reference, the following bibliography is presented for those readers wishing to acquaint themselves further with the subject.

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